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F. W. DYSON, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Major Archibald S. Dunlop, Royal Field Artillery, Chief Instructor in Topography, Royal Military Academy, Woolwich;

Le Comte A. de Gramont, Docteur ès Sciences Physiques, 179 rue de l'Université, Paris, VII.; and

Patrick Henry Hepburn, LL.B. (Lond.), 49 Downshire Hill, Hampstead, N.W.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Arthur Gilbert Fellows, Schoolmaster, 18 Wellington Road, Watford, Herts (proposed by Samuel Fellows);

Charles Whitsey Franklin, 15 The Grove, Swansea, South Wales (proposed by James Weir);

David Llewellyn Howe, 12 Leigh Road, Clifton, Bristol (proposed by E. Fawcett White); and

James Mason, M.A., B.Sc., Teacher, Sutherland Technical School, Golspie, Scotland (proposed by J. Halm).

The following were proposed by the Council as Associates of the Society:—

Baron Roland Eötvös, Professor of Physics in the University, Budapest, Hungary.

Annibale Riccò, Director of the Observatory, Catania, Sicily.

Seventy presents were announced as having been received since the last meeting, including, amongst others:—

F. K. McClean, Report of the Solar Eclipse Expedition to Port Davey, Tasmania, May 1910, presented by the Author; and 22 charts of the Astrographic Chart of the Heavens, presented by the Observatory, San Fernando.

On the Oscillating Orbits about the Triangular Equilibrium Points in the Problem of three Bodies. By Ernest W. Brown, Sc.D., F.R.S.

1. It is well known that when three bodies are placed at the corners of an equilateral triangle and started with suitable velocities they will maintain their relative configuration under the Newtonian law of attraction. It is further known that, provided the masses of the three bodies satisfy a certain relation, the motion is stable. The object of this paper is an examination of a particular case when this relation between the masses is not satisfied. I shall show that closed orbits about these relative equilibrium points still exist, but that such orbits cannot be of infinitesimal dimensions.

2. The limitations are the same as those adopted by Sir George Darwin in his papers on periodic orbits, and in my previous paper in the *Monthly Notices*, lxxi. p. 438, on the same subject. The mass of one body, the Sun, is taken for unity; that of the second body, Jove, is denoted by m; while that of the third body, the asteroid, is infinitesimal. Jove moves round the Sun in a circle of radius unity with constant angular velocity $n=(1+m)^{\frac{1}{2}}$, and the three bodies move in a fixed plane.

The equations of motion referred to moving axes of which the x-axis passes through Jove and the origin is the Sun are

$$\ddot{x}-2n\dot{y}=\frac{\partial\Omega}{\partial x},$$

$$\ddot{y}+2n\dot{x}=\frac{\partial\Omega}{\partial y},$$

where

$$\Omega=\frac{2}{r}+r^2+m\left(\frac{2}{\rho}+\rho^2\right), \quad . \quad . \quad (1)$$

ρ being the distance of the asteroid from Jove and $r^2=x^2+y^2$.

Let us adopt Hill's transformation by putting

$$u=x+\iota y, \qquad s=x-\iota y, \qquad \frac{d}{dt}=\iota D.$$

The equations of motion are then represented by the single equation

$$D^2u + 2nDu + 2\frac{\partial \Omega}{\partial s} = 0 \quad . \quad . \quad . \quad (2)$$

If we turn the axes round through a constant angle $\frac{1}{3}\pi - \beta$, and allow the letters x, y, u, s to represent coordinates referred to the new axes, the equation of motion (2) will be unaltered.

Let

$$u_0 = \exp. \tfrac{1}{3}\pi\iota, \qquad s_0 = \exp. -\tfrac{1}{3}\pi\iota,$$

be the coordinates of the upper equilibrium point referred to the old axes. The coordinates of the same point referred to the new axes are then exp. $\beta\iota$, exp. $-\beta\iota$.

Put

$$u = e^{\beta\iota} + u_1, \qquad s = e^{-\beta\iota} + s_1,$$

and expand (2) in powers of u_1, s_1. We obtain

$$(D^2 + 2nD + a_1)u_1 + b'_1 s_1 + \tfrac{1}{2}b_2 u_1^2 + b'_2 u_1 s_1 + \tfrac{1}{2}c'_2 s_1^2 + \tfrac{1}{3}a_3 u_1^3 + b_3 u_1^2 s_1 + a_3 u_1 s_1^2 + \tfrac{1}{3}c'_3 s_1^3 + \ldots = 0, \quad (3)$$

in which a_1, b_1, . . . are the values of certain derivatives of 2Ω at the equilibrium point. This equation is to be solved by continued approximation.

3. Suppose that we neglect all powers of u_1, s_1 but the first. The equation becomes linear, and if we put $u_1 = A_1 e^{\lambda\iota t}$, the values of λ are determined by

$$\lambda^4 - (1+m)\lambda^2 + \tfrac{27}{4}m = 0.$$

The roots are real, and all the solutions are periodic provided $(1+m)^2 > 27m$. If this inequality is not satisfied the roots are complex. Suppose, without making any supposition as to the value of m or the reality of the roots, that we proceed to higher approximations. The equation for λ then becomes of the form

$$\lambda^4 - (1+m)\lambda^2 + \tfrac{27}{4}m = ka^2, \quad . \quad . \quad . \quad (4)$$

where a is an arbitrary constant proportional to the linear dimensions of the oscillation, and k is a function of m only. The roots of this equation will be real provided

$$\tfrac{1}{4}(1+m)^2 - \tfrac{27}{4}m + ka^2 > 0.$$

Hence real values of λ will still be possible even if $(1+m)^2 < 27m$, provided k is positive and a sufficiently large. There will then exist closed orbits round the equilibrium point, but such orbits cannot be infinitely small. The first point of the following investigation is a determination of the value of k.

4. Most of the coefficients a_1, b_1, . . . are complex quantities; the notation adopted is such that an accented symbol is conjugate to the same symbol unaccented. If we change the sign of $\sqrt{-1}$ accented letters become unaccented, and *vice versâ*, D changes sign, and u, s are interchanged. In particular

$$a_1 = 2\frac{\partial^2 \Omega}{\partial u \partial s}, \qquad b'_1 = 2\frac{\partial^2 \Omega}{\partial s^2}, \qquad b_1 = 2\frac{\partial^2 \Omega}{\partial u^2},$$

where $u = e^{\beta\iota}$, $s = e^{-\beta\iota}$. Hence a_1 is real, while b_1, b'_1 are complex. In order to facilitate the work, the angle β will be so chosen that b_1, b'_1 are real, and therefore equal. It will then follow that $\partial^{m+n}\Omega/\partial u^m \partial s^n$ is real when $|m-n| = 2$. Hence a_1, b_1, a_3, b_3 are real, but b_2, c_2, c_3 are complex.

We must first determine β by the condition just laid down. Since u_0, = exp. $\frac{1}{3}\pi\iota$, is the equilibrium point referred to the old axes,

$$\tfrac{1}{2}b_1 = \frac{\partial^2 \Omega}{\partial u_0^{\,2}} \text{ exp. } (\tfrac{2}{3}\pi\iota - 2\beta\iota).$$

Also $r_0^{\,2} = u_0 s_0 = 1$, $\rho_0^{\,2} = (1-u_0)(1-s_0) = 1$. Hence

$$\frac{\partial^2 \Omega}{\partial u_0^{\,2}} = \tfrac{3}{4}\frac{s_0^{\,2}}{r_0^{\,5}} + \tfrac{3}{4}m\frac{(1-s_0)^2}{\rho_0^{\,5}} = -\tfrac{3}{4}u_0 + \tfrac{3}{4}m u_0^{\,2}.$$

Therefore

$$\tfrac{1}{2}b_1 = \tfrac{3}{4}(1 - mu_0)e^{-2\beta\iota}.$$

The condition $b_1 = b'_1$ gives

$$e^{2\beta\iota} = \frac{1-mu_0}{\sqrt{1-m+m^2}}, \qquad \tan 2\beta = \frac{m\sqrt{3}}{2-m},$$

where the small value of β is chosen; hence the new x-axis very nearly goes through the equilibrium point and the Sun when m is small. This is, of course, the same result as that obtained in my previous paper on this subject in the *Monthly Notices*.

5. The other derivatives of Ω, and thence the coefficients in (3), can now be computed. The results are as follows:—

$$a_1 = 2\frac{\partial^2 \Omega}{\partial u \partial s} = \tfrac{3}{2}(1+m)$$

$$b_1 = 2\frac{\partial^2 \Omega}{\partial u^2} = \tfrac{3}{2}\sqrt{1-m+m^2},$$

$$b_2 = 2\frac{\partial^3 \Omega}{\partial u^2 \partial s} = -\tfrac{3}{4}(1+ms_0)e^{\beta\iota},$$

$$c_2 = 2\frac{\partial^3 \Omega}{\partial u^3} = -\tfrac{15}{4}(1-m)e^{3\beta\iota},$$

$$a_3 = \frac{\partial^4 \Omega}{\partial u^3 \partial s} = \tfrac{15}{16}\sqrt{1-m+m^2},$$

$$b_3 = \frac{\partial^4 \Omega}{\partial u^2 \partial s^2} = \tfrac{9}{16}(1+m),$$

$$c_3 = \frac{\partial^4 \Omega}{\partial u^4} = \tfrac{105}{16}(1-ms_0)e^{4\beta\iota},$$

in which the values of u, s at the equilibrium point have of course been substituted.

6. It is assumed that a solution of (3) can be obtained in a series proceeding along powers of an arbitrary parameter and cosines and sines of multiples of an angle λt. It is not necessary to put the parameter in evidence; a suffix will denote the power of this parameter supposed to be attached to any coefficient. It is evident that the coefficients of cos λt, sin λt will contain only odd powers of this parameter, beginning with the first; that the coefficients of cos $2\lambda t$, sin $2\lambda t$ will contain only even powers, beginning with the second; and so on. Assume therefore that

$$u_1 = (P_1+P_3)e^{\lambda t\iota} + (Q_1+Q_3)e^{-\lambda t\iota} + P_2 e^{2\lambda t\iota} + Q_2 e^{-2\lambda t\iota} + R_2 \quad (5)$$

with the corresponding value for s in which ι changes sign and the coefficients are accented to denote the conjugate complexes. The terms in $e^{\pm 3\lambda t\iota}$ will not be required; with this limitation, (5) is the value of u to the third order inclusive.

Substitute (5) in (3) and equate to zero the coefficient of $e^{\lambda t\iota}$. Also equate to zero the coefficient of $e^{-\lambda t\iota}$ and then change the sign of ι. We obtain

$$\left.\begin{aligned}(P_1+P_3)(\lambda^2+2n\lambda+a_1)+(Q'_1+Q'_3)b_1+S_3 &= 0,\\ (Q'_1+Q'_3)(\lambda^2-2n\lambda+a_1)+(P_1+P_3)b_1+T_3 &= 0,\end{aligned}\right\} \quad (6)$$

where S_3, T_3 represent certain terms of the third order at the lowest.

Retaining only terms up to the third order inclusive, and using the values of a_1, b_1 given in No. 5, we deduce, since $n^2 = 1+m$,

$$\left(\lambda^2 - \frac{n^2}{2}\right)^2 - \tfrac{1}{4}(1+m)^2 + \tfrac{27}{4}m = b_1 \frac{P_1S_3 + Q'_1T_3}{P_1Q'_1}. \quad (7)$$

According to No. 3, it is the right-hand member of this equation which is to be computed.

Now the case in which $(1+m)^2 < 27m$ is under consideration. We further require that λ^2 shall be real if possible. Hence we must regard $\frac{1}{4}(1+m)^2 - \frac{27}{4}m$ as of the second order with respect to the parameter, and therefore in a first approximation this quantity may be placed equal to zero. Then $\lambda = \pm n/\sqrt{2}$. It is immaterial which of these values we choose: take $\lambda = +n/\sqrt{2}$.

We now have with this value of m,

$$a_1 = \tfrac{3}{2}n^2, \quad b_1 = n^2\sqrt{2}.$$

Neglecting terms of the third order in (6), we obtain the first approximation. Obviously P_1, Q_1 are real, so that $P'_1 = P_1$, $Q'_1 = Q_1$. Hence the first approximation is given by

$$(1+m)^2 = 27m, \quad \lambda = n/\sqrt{2}, \quad Q_1 = -(\sqrt{2}+1)P_1 \qquad (8)$$

7. The second approximation is obtained by equating to zero the coefficients of $e^{\sigma t\iota}$, $e^{\pm 2t\iota}$, after the substitution of (5) in (3). This procedure gives

$$a_1 R_2 + b_1 R'_2 + S_2 = 0, \quad . \quad . \quad . \qquad (9)$$

$$\left.\begin{aligned} (4\lambda^2 + 4n\lambda + a_1)P_2 + b_1 Q'_2 + T_2 &= 0, \quad . \\ (4\lambda^2 - 4n\lambda + a_1)Q'_2 + b_1 P_2 + U_2 &= 0, \quad . \end{aligned}\right\} \qquad (10)$$

where

$$S_2 = (b_2 + c'_2)P_1 Q_1 + b'_2(P_1^2 + Q_1^2), \quad . \quad . \qquad (11)$$

$$\left.\begin{aligned} 2T_2 &= b_2 P_1^2 + c'_2 Q_1^2 + 2b'_2 P_1 Q_1, \quad . \\ 2U_2 &= b'_2 Q_1^2 + c_2 P_1^2 + 2b_2 P_1 Q_1, \quad . \end{aligned}\right\} \qquad (12)$$

with six similar equations obtained by interchanging the accents.

8. The values of S_3, T_3 defined in No. 6 are as follows:—

$$\begin{aligned} S_3 = {} & b_2(P_1 R_2 + Q_1 P_2) + b'_2(P_1 R'_2 + Q_1 R_2 + P_1 P_2 + Q_1 Q'_2) \\ & + c'_2(P_1 Q'_2 + Q_1 R'_2) + a_3(P_1^2 Q_1 + Q_1^3 + 2P_1^2 Q_1) \\ & + b_3(P_1^3 + 2P_1 Q_1^2) + c'_3 P_1 Q_1^2, \\ T_3 = {} & b'_2(P_1 Q'_2 + Q_1 R'_2) + b_2(P_1 R'_2 + Q_1 R_2 + P_1 P_2 + Q_1 Q'_2) \\ & + c_2(P_1 R_2 + Q_1 P_2) + a_3(P_1 Q_1^2 + P_1^3 + 2P_1 Q_1^2) \\ & + b_3(Q_1^3 + 2P_1^2 Q_1) + c'_3 P_1^2 Q_1. \end{aligned}$$

Hence, with the assistance of (11), (12),

$$\begin{aligned} P_1 S_3 + Q_1 T_3 = {} & R_2 S'_2 + R'_2 S_2 + 2P_2 T'_2 + 2Q'_2 U'_2 \\ & + P_1^2 Q_1^2(c_3 + c'_3 + 2b_3) + (P_1^2 + Q_1^2)\{4a_3 P_1 Q_1 + b_3(P_1^2 + Q_1^2)\}. \end{aligned}$$

We can eliminate R_2, R'_2, P_2, Q'_2 by means of (9), (10), and obtain the required function expressed in the form

$$\begin{aligned} P_1 S_3 + Q_1 T_3 = {} & -\tfrac{1}{2}\frac{(S_2 + S'_2)^2}{a_1 + b_1} + \tfrac{1}{2}\frac{(S_2 - S'_2)^2}{a_1 - b_1} \\ & + \frac{2}{F}\Big[b_1(U_2 T'_2 + U'_2 T_2) - T_2 T'_2(4\lambda^2 - 4n\lambda + a_1) - U_2 U'_2(4\lambda^2 + 4n\lambda + a_1) \Big] \\ & + 4a_3 P_1 Q_1(P_1^2 + Q_1^2) + b_3(P_1^2 + Q_1^2)^2 + (c_3 + c'_3 + 2b_3)P_1^2 Q_1^2, \end{aligned}$$

where

$$F = (4\lambda^2 + a_1)^2 - 16n^2\lambda^2 - b_1^2 = \tfrac{9}{4}n^2,$$

after the substitution of the values (8) given by the first approxima-

tion. It may be noted that though $a_1 - b_1$ is implicitly divisible by m, $S_2 - S'_2$ is also divisible by m.

9. Every term in the long expression just given is of the fourth order, and therefore we can use the values given by the first approximation to compute S_2, T_2, etc. I shall not give the details of the long and tedious numerical computation necessary to obtain its value. The parts containing S, U and T, P and Q were obtained separately and gave

$$\left.\begin{aligned} P_1S_3 + Q_1T_3 &= (-\tfrac{823}{192} - \tfrac{14479}{34560} + \tfrac{10115}{128})n^2P_1^2Q_1^2 \\ &= -\tfrac{59}{108}n^2P_1^2Q_1^2. \end{aligned}\right\} \quad (13)$$

Since $P_1^2 + Q_1^2 = -2\sqrt{2}\, P_1Q_1$ from (8), $Q'_1 = Q_1$, and $b_1 = n^2\sqrt{2}$, we obtain from (7)

$$(\lambda^2 - \tfrac{1}{2}n^2)^2 = \tfrac{1}{4}n^4 \left\{ 1 - 27\frac{m}{(1+m)^2} + \tfrac{59}{54}(P_1^2 + Q_1^2) \right\}.$$

Hence the roots of the period equation will be real provided

$$P_1^2 + Q_1^2 > \tfrac{54}{59} \left| 1 - \frac{27m}{(1+m)^2} \right|,$$

whatever be the sign of $(1+m)^2 - 27m$.

10. This result will not be sufficient to settle the question unless $P_1^2 + Q_1^2$ is small, that is, unless m is approximately equal to $1/25$. The value of $P_1^2 + Q_1^2$ must be compared with the distance between the Sun and Jove, that is, with unity. In the first approximation this quantity is half the sum of the squares of the semi-axes of the small ellipse described about the equilibrium point. Let us take $\alpha = \sqrt{\tfrac{1}{2}P_1^2 + \tfrac{1}{2}Q_1^2}$ as the measure of the linear dimensions of the orbit. The following table then gives the minimum value of α for different values of m,

$$m = \tfrac{1}{25}, \quad \tfrac{1}{24}, \quad \tfrac{1}{22}, \quad \tfrac{1}{20}.$$
$$\text{min. } \alpha = 0, \quad \cdot130, \quad \cdot237, \quad \cdot320.$$

For values of m greater than $1/20$ it will be necessary to go to higher powers of P_1, Q_1 or to abandon the method. We may however conclude that orbits round the equilibrium point exist for values of m greater than $1/25$, and that when m is near this value there are two such orbits which spring from a single orbit in which the period is $2\pi\sqrt{2}/n$.

11. *The constant of Energy.*—Let Ω be expanded in powers of u_1, s_1, so as to give $\Omega = \Sigma\Omega_p$, where Ω_p denotes the terms of order p with respect to u_1, s_1. The energy equation is then

$$Du_1.Ds_1 = C - 2\Omega = C - 2\Sigma\Omega_p \quad . \quad . \quad (14)$$

At the equilibrium point we have $2\Omega_0 = 3 + 3m$ and $\Omega_1 = 0$.

Also multiplying the equation (3) and its conjugate by s_1, u_1 respectively and adding, we can write the result in the form

$$D^2(u_1 s_1) - 2Du_1.Ds_1 + 2n(s_1 Du_1 - u_1 Ds_1) = -2\Sigma p\,\Omega_p \qquad (15)$$

where $p = 2, 3, \ldots \infty$. From these equations we deduce

$$2C = 4Du_1.Ds_1 - 2n(s_1 Du_1 - u_1 Ds_1) - 2\Sigma(p-2)\,\Omega_p - D^2(u_1 s_1) + 6 + 6m.$$

In order to find C, substitute the values of u_1, s_1 previously obtained. Then $u_1 s_1$ is real and contains only periodic terms; therefore $D^2(u_1 s_1)$ contributes nothing to C. If we retain only the terms of the lowest order (the second) in C, $\Sigma(p-2)\,\Omega_p = 0$. With the first approximation we then find

$$C = -2\lambda^2(P_1^2 + Q_1^2) - \lambda n(P_1^2 - Q_1^2) + 3 + 3m.$$

In this approximation, $\lambda = n/\sqrt{2}$, $Q_1 = -(\sqrt{2}+1)P$. Hence $C = 3 + 3m$. Its value as far as the terms of the second order inclusive is independent of the size of the orbit.

12. Consider the equation (14). The coefficients of the terms in the right-hand member do not contain λ explicitly, while the left-hand member will contain it only as a factor λ^2. If then we substitute the value (5) in this equation, we shall obtain C in the form, to the fourth order inclusive,

$$C = 3 + 3m + k(P_1^2 + Q_1^2)^2 + k'(P_1^2 + Q_1^2)(\lambda^2 - \tfrac{1}{2}n^2),$$

where k, k' are numerical coefficients to the degree of approximation adopted. In the limiting orbit $\lambda^2 = \frac{1}{2}n^2$. If then C_0 be the value of C corresponding to the limiting orbit, we shall have $C > C_0$ in one of the two orbits which spring from this limiting orbit and $C < C_0$ in the other. I have not computed the coefficients k, k', except in so far as it was necessary to prove that k is negative. Hence C increases with the period. This is in accordance with a similar result obtained, when m was very small, in my previous paper.

13. We have hitherto considered the two orbits separately, that is to say, of the four arbitrary constants present in the general solution, two have been placed equal to zero, and we have then made a second and a third approximation with these two constants still zero; this has been done for each of the two pairs of solutions. I shall now find the orbit when all the constants are retained; this orbit will not be re-entrant in general.

14. Let λ^2 denote that value which is greater than $n^2/2$, and μ^2 that value which is less than $n^2/2$. The complete first approximation will then be denoted by

$$u_1 = P_1 e^{\lambda i t} + Q_1 e^{-\lambda i t} + p_1 e^{\mu i t} + q_1 e^{-\mu i t}.$$

In the coefficients of this first approximation we have

$$\lambda^2 = \mu^2, \qquad Q_1/P_1 = q_1/p_1 = -\sqrt{2} - 1.$$

15. In the second approximation we shall have the terms corresponding to $e^{\mp 2\lambda\iota t}$, e^{0t} arising from P_1, Q_1 as before; a precisely similar set of terms arising from p_1, q_1, with coefficients denoted by p_2, q_2, r_2; and, in addition, a new set of terms involving the arguments $(\pm\lambda \pm \mu)\iota t$. Let these additional terms be denoted by

$$u_1 = L_2 e^{(\lambda+\mu)\iota t} + M_2 e^{-(\lambda+\mu)\iota t} + l_2 e^{(\lambda-\mu)\iota t} + m_2 e^{-(\lambda-\mu)\iota t}.$$

On substituting this with the complete first approximation in (3) and equating to zero the coefficients of the terms with exponents $\pm(\lambda+\mu)\iota t$, we obtain

$$\{(\lambda+\mu)^2 + 2n(\lambda+\mu) + a_1\}L_2 + b_1 M'_2 + b_2 P_1 p_1 + c'_2 Q_1 q_1 + b'_2(P_1 q_1 + Q_1 p_1) = 0,$$
$$\{(\lambda+\mu)^2 - 2n(\lambda+\mu) + a_1\}M'_2 + b_1 L_2 + b'_2 Q_1 q_1 + c_2 P_1 p_1 + b_2(P_1 q_1 + Q_1 p_1) = 0.$$

When we solve for L_2, M'_2, the common divisor in the results is

$$\{(\lambda+\mu)^2 + a_1\}^2 - 4n^2(\lambda+\mu)^2 - b_1^2.$$

This divisor is not small; and as all the terms are of the second order, and further, as λ, μ only differ from one another by small quantities, we can place $\lambda = \mu$ in these equations. They then become the same as those for the arguments $\pm 2\lambda$, that is, the same as (10) if we replace in (12)

$$P_1^2, \quad Q_1^2, \quad P_1 Q_1 \text{ by } 2P_1 p_1, \quad 2Q_1 q_1, \quad P_1 q_1 + Q_1 p_1, \qquad (16)$$

respectively. Hence, since $P_1 q_1 = Q_1 p_1$,

$$\frac{L_2}{P_1 p_1} = \frac{2P_2}{P_1^2}, \quad \frac{M_2}{P_1 p_1} = \frac{2Q_2}{P_1^2}, \qquad (17)$$

with similar results for the conjugate coefficients.

The terms with exponents $\pm(\lambda-\mu)\iota t$ are treated in the same way. The common divisor in the solution is

$$\{(\lambda-\mu)^2 + a_1\}^2 - 4n^2(\lambda-\mu)^2 - b_1^2,$$

which, when $\lambda = \mu$, becomes $a_1^2 - b_1^2$. The equations for l_2, m'_2 become the same as those for R_2, R'_2, numbered (9) above, if we replace, in (11),

$$P_1^2, \; Q_1^2, \; P_1 Q_1 \quad \text{by} \quad P_1 p_1, \; Q_1 q_1, \; \tfrac{1}{2}(P_1 q_1 + Q_1 p_1),$$

respectively. Therefore

$$\frac{l_2}{P_1q_1}=\frac{m_2}{P_1q_1}=\frac{R_2}{P_1^2}, \qquad \frac{m'_2}{P_1q_1}=\frac{l'_2}{P_1q_1}=\frac{R'_2}{P_1^2} \quad . \quad . \quad (18)$$

16. We have now to find the additional terms of the third order. It is necessary to obtain the further approximations to λ^2, μ^2 separately; let us choose the former. We have to find the additional terms produced in u_1^2, s_1^2, u_1s_1 when we substitute the complete second approximation in (3). The additional terms which will give arguments $\pm\lambda\iota t$ are first those arising from the products of

$$u_1=L_2e^{(\lambda+\mu)\iota t}+M_2e^{-(\lambda+\mu)\iota t}+l_2(e^{(\lambda-\mu)\iota t}+e^{-(\lambda-\mu)\iota t}),$$
$$s_1=L'_2e^{-(\lambda+\mu)\iota t}+M'_2e^{(\lambda+\mu)\iota t}+l'_2(e^{(\lambda-\mu)\iota t}+e^{-(\lambda-\mu)\iota t}),$$

with

$$u_1=p_1e^{\mu\iota t}+q_1e^{-\mu\iota t},$$
$$s_1=p_1e^{-\mu\iota t}+q_1e^{\mu\iota t}.$$

Let us compare the results obtained by the combinations of the arguments $\pm(\lambda+\mu)\iota t$ with $\mp\mu\iota t$, with those produced by the combinations $\pm2\lambda\iota t$ with $\mp\lambda\iota t$. It is evident that they are precisely similar. Hence these new terms in S_3, T_3 are obtained by replacing Q_1P_2, P_1P_2, . . . by q_1L_2, p_1L_2, . . . , that is, on account of (17), by $2p_1q_1P_2/P_1$, $2p_1^2P_2/P_1$, ... Hence the new terms in $(P_1S_3+Q_1T_3)$ due to the exponents $\pm(\lambda+\mu)\iota t$ are obtained by replacing in the terms due to U_2, T_2, U'_2, T'_2 the factor $P_1^2Q_1^2$ by $2P_1Q_1p_1q_1$. They therefore give, from (13),

$$-2\frac{14479}{3456}n^2P_1Q_1p_1q_1 \quad . \quad . \quad . \quad (19)$$

The terms arising from the exponents $\pm(\lambda-\mu)\iota t$ similarly give

$$-\frac{823}{192}n^2P_1Q_1p_1q_1 \quad . \quad . \quad . \quad . \quad (20)$$

We have also additional terms in u_1^2, s_1^2, u_1s_1, arising from the products of

$$u=r_2, \quad s=r'_2,$$

with

$$u=P_1e^{\lambda\iota t}+Q_1e^{-\lambda\iota t},$$
$$s=P_1e^{\lambda\iota t}+Q_1e^{-\lambda\iota t}.$$

These contribute

$$-\frac{823}{192}n^2P_1Q_1p_1q_1$$

to $P_1S_3+Q_1T_3$.

Finally we have the new terms in u_1^3, $u_1^2s_1$, $u_1s_1^2$, s_1^3 contributed

by the squares and products of the solution of the first order with exponents $\pm \mu\iota t$, with the solution of the first order with exponents $\pm \lambda\iota t$. These contribute

$$+ 2 \frac{1015}{128} n^2 P_1 Q_1 p_1 q_1 .$$

The complete addition to $P_1 S_3 + Q_1 T_3$ is therefore

$$- 2 \frac{59}{108} n^2 P_1 Q_1 p_1 q_1 .$$

17. The new value of λ^2 immediately follows. The value of μ^2 is similarly obtained. These values are given by

$$\lambda^2 = \tfrac{1}{2} n^2 + \tfrac{1}{2} n^2 \left\{ 1 - 27 \frac{m}{(1+m)^2} + \frac{59}{54}(P_1^2 + Q_1^2) + \frac{59}{27}(p_1^2 + q_1^2) \right\}^{\frac{1}{2}},$$

$$\mu^2 = \tfrac{1}{2} n^2 - \tfrac{1}{2} n^2 \left\{ 1 - 27 \frac{m}{(1+m)^2} + \frac{59}{54}(p_1^2 + q_1^2) + \frac{59}{27}(P_1^2 + Q_1^2) \right\}^{\frac{1}{2}}.$$

Hence the solutions are stable. For infinitesimal deviations from the first solution, we neglect $p_1^2 + q_1^2$, and the second solution is real so long as the first periodic solution exists, that is, so long as λ^2 is real. The same is true of the second solution when μ^2 is real and we neglect $P_1^2 + Q_1^2$.

18. The two solutions coincide when the portions under the square roots are zero; these conditions demand that $P_1 = p_1$, $Q_1 = q_1$, and that

$$1 - 27 \frac{m}{(1+m)^2} + 3 \tfrac{59}{54}(P_1^2 + Q_1^2) = 0 .$$

This appears to be at variance with the previous condition which demanded that

$$1 - 27 \frac{m}{(1+m)^2} + \tfrac{59}{54}(P_1^2 + Q_1^2) = 0 ,$$

even when we take account of the fact that when $\lambda = \mu$, $P_1 = p_1$, $Q_1 = q_1$, our first approximation reduces to $\mu_1 = 2P_1 e^{\lambda t} + 2Q_1 e^{-\lambda t}$. The difficulty arises from the fact that the form of the solution is not continuous at the value $\lambda = \mu$. There are terms of the third order with the arguments $\pm(2\lambda - \mu)\iota t$, $\pm(2\mu - \lambda)\iota t$ which do not affect the coefficients P_3, Q_3 as long as $\lambda \neq \mu$; but when $\lambda = \mu$ these terms become confounded with the terms with arguments $\pm \lambda \iota t$ and consequently modify P_3, Q_3, and the apparent value of λ^2. The treatment of the case $\lambda^2 = \mu^2 = n^2/2$ must therefore be the same as that of the periodic orbit in which we put either $p_1 = q_1 = 0$ or $P_1 = Q_1 = 0$.

19. It has been found that $Q_1 = -(\sqrt{2} + 1) P_1 = -2{\cdot}414 P_1$

in the first approximation when $m = 1/25$. Now the ellipse which constitutes the first approximation is determined by eliminating λ from

$$u_1 = P_1 e^{\lambda t_i} + Q_1 e^{-\lambda t_i},$$

or between

$$x_1 = (P_1 + Q_1) \cos \lambda t, \quad y_1 = (P_1 - Q_1) \sin \lambda t.$$

Hence the ratio of the axis of the small ellipse is

$$\frac{P_1 + Q_1}{P_1 - Q_1} = -(\sqrt{2} - 1) = -.414.$$

The shorter axis of the ellipse is therefore along the x-axis, that is, very nearly along the line joining the Sun and Jove. A knowledge of the maximum and minimum diameters when we proceed to higher approximations is perhaps desirable, but the necessary arithmetical work is somewhat extensive. In view of the fact that this series of orbits is shown to exist only for a limited range of values of m and of P_1, Q_1, a more complete arithmetical computation may properly await the need, if such should arise, for more detailed knowledge of the orbits.

20. The extension of these orbits to cases in which P_1, Q_1 are larger, and to cases in which m becomes greater than about $1/20$, cannot be made by the methods of this paper. If the continuity of the orbits follows the same general lines as for the cases when m is very small, we should expect the solutions with the longer period to develop into horse-shoe orbits, as shown in my previous paper. To prove the existence of such orbits it will probably be necessary to trace them by some arithmetical process like that adopted by Sir George Darwin in his investigations on periodic orbits. In view of the changes which have been shown to occur with a change in the value of m from a quantity of the order $1/1000$ to one of the order $1/20$, one hesitates to affirm that the family existing in the former case will still exist with the same general series of changes when m becomes greater than $1/20$. At the same time, we still have the two pairs of solutions near the equilibrium point in opposition to Jove, and they respectively remain elliptic and hyperbolic. A definite basis for further investigation therefore exists.

New Haven, Conn.:
1911 *February* 15.

On Osculating Elements. By R. T. A. Innes.

In my paper "On the Mean or Perihelion Distances of Comets" which appeared in the *Monthly Notices* for last December, the proposition was advanced that the elliptic equation between the mean motion and the semi-axis major does not hold in the solar system, and, as a consequence, that the usual formulæ connecting the perihelion distance and true anomaly of a comet are not exact.

In the case of a comet travelling in a parabolic orbit, the factor k in the equation

$$\frac{k(t-\mathrm{T})}{\sqrt{2}q^{\frac{3}{2}}} = \tan\tfrac{1}{2}\nu + \tfrac{1}{3}\tan^3\tfrac{1}{2}\nu$$

should be replaced by

$$k\sqrt{1+\Sigma m},$$

in which Σm is the sum of the masses of the planets.

We are aware that when a planet is travelling through the solar system it is acted on by forces due to the Sun and the planets, but the exigencies of calculation require that at first the solar action should alone be considered. It is, however, impossible to determine how much of each element is due to solar action alone unless the general perturbations are computed. In particular, n, the mean motion, which is derived solely from observations, includes a constant part due to planetary action. The only consistently logical astronomer, in dealing with the constants introduced by integration as compared with those derived by observation, is Hansen. He shows that a wide latitude is permissible, provided always that the adjustment is finally made. This adjustment is not made in the methods of special perturbations used universally to-day with comets and minor planets. The procedure is to start with an osculating orbit derived from observations, using, where possible, a mean motion depending on an extended series of observations; with these elements the special perturbations are built up and applied. In consequence, the constant parts of the perturbations are not separately determined. There is, however, no doubt that the errors thus introduced are negligible, with one exception. One cannot, from the osculating or mean value of n, derive the true mean distance by means of the elliptic equation. The error which will be introduced in doing so will frequently be large. Gauss furnishes us with a ready-made example, which I beg to be allowed to quote.

In his work on Pallas (Gauss, vii., 1906, p. 527), he gives the osculating values of n for seven oppositions, and the elliptic values of a which result. To find the mean value of the semi-axis major he applies a correction to the figures deduced by means of $a^3n^2 = k^2$. Gauss's figures are as follows:—

n.	Log Elliptic a.	δ Log a.	Mean Value of Log Semi-axis Major.
770·7240	0·442,0718	−5648	0·442,6366
3919	1966	−4381	6347
1365	2926	−3424	6350
4995	1562	−4729	6291
1737	2786	−3516	6302
768·2675	9961	+3776	6185
8375	7813	+1683	6130

It will be seen that whilst the incomplete or elliptic log semi-axis major varies from 0·442,0718 to 0·442,9961, the mean value only varies from 0·442,6130 to 0·442,6366.

It is quite true that Hansen consistently uses the equation

$$a^3n^2 = a_0{}^3n_0{}^2,$$

throwing the change necessary on to ν in the equation

$$r = r(1+\nu).$$

If, however, there is no intention of determining the constant in ν, it is better to throw it on to a. In fact it is always better to modify a, and this is the procedure followed by Newcomb and Hill in their planetary theories. The object of my two papers is to recommend that it be done universally. If there is no intention of computing the general perturbations, the resulting a will be nearer the truth than the elliptic a, and so form a securer basis for the special perturbations. If the general perturbations are computed, then the constant introduced into ν is to be modified. In section 30 of his *Ermittelung der Absoluten Störungen in Ellipsen von beliebiger Excentricität*, etc., 1843, Hansen indicates how the constants are to be modified in the different cases which arise.

In Messrs. Cowell and Crommelin's work on Halley's Comet they pointed out that if the perturbing function is divided into two portions V_3 and V_4, the part V_3 permits of finite integration. That this is so seems to have been first noticed by Hansen, who frequently refers to it. (See *loc. cit.*, sections 4 and 69, *Auseinandersetzung*, part ii., section 3; and *Störungen . . . des Jupiters*, 1875, p. 304.) This principle has considerable bearing on my argument, and attention may be directed to the results of Hansen's integration of V_3 (*Auseinandersetzung*, part ii., par. 34, p. 60), where he obtains remarkably simple expressions for ndz and ν, and shows how the constants are to be determined. His result is that the only constant to be adopted is $+\frac{1}{3}m'$ in the expression for ν. This agrees very closely with the procedure I recommend. If we replace Hansen's

$$\nu \text{ by } \tfrac{1}{3}m' + \nu_1,$$

in which ν_1 is made up of periodic terms, we have

$$\underline{r} = r(1 + \tfrac{1}{3}m' + \nu_1) \quad . \quad . \quad . \quad . \quad (A)$$

If, however, we start with the equation

$$r = \frac{a(1 + \frac{1}{3}m')(1 - e^2)}{1 + e\cos f}$$

we can replace (A) by

$$\underline{r} = r(1 + \nu_1).$$

These formulæ apply to planets and comets when outside of the orbit of the body whose mass is m'. (See also Hansen, *loc. cit.*, par. 36.)

I have dealt with this subject from another point of view in the *South African Journal of Science* for last January (see p. 108), wherein the following formulæ are given. The comet being distant from the planet's orbit, we have the two cases

	Comet inside Planet's Orbit.	Comet outside Planet's Orbit.
$\delta \log a$	$-\frac{1}{6}m'\alpha^3$	$+\frac{1}{3}m'(1 + \frac{3}{4}\alpha^2)$

in which α is always less than unity.

These simple, but be it understood only approximate, formulæ convey the sense of Dr. Crommelin's comments on page 131 of the *Monthly Notices* for December last. It will be obvious that the diminution of attraction when the comet is inside a planet's orbit is less than the increase when it is outside, so that the consideration of Dr. Crommelin's table (p. 131) alone might mislead.

But I freely acknowledge that my correction to Messrs. Cowell and Crommelin's mean distance of Halley's Comet given in my last paper may be, and probably is, greatly in error. My correction applies not to an osculatory n, but to the mean n over a complete revolution. I cannot, however, grant that my correction necessitates a change of 18·8 days in the date of perihelion passage in 1910, because if my value of a had been used in deriving the comet's elements (still retaining the n used by Messrs. Cowell and Crommelin), the other elements (particularly the eccentricity) would have been changed. In fact, many ellipses will pass through the two places of departure actually used, or even represent a somewhat extended series of observations within finer limits than the accuracy of observations of comets requires.

Johannesburg:
1911 *February* 15.

Note on Mr. Innes' Paper. By A. C. D. Crommelin, D.Sc.

I should like to add a short note to Mr. Innes' final paragraph. I cordially agree with the sentence, "in fact many ellipses will pass through . . . . comets requires." I have recently urged the same point myself in reference to attempts that have been made to obtain the mean motion of Halley's Comet in 1910 from observations at the present return alone, without reference to previous returns. The correct procedure seems to be that followed by Westphalen in discussing the 1835 return, viz. to take the mean motion n given by theory, and to find all the elements with small corrections involving δn, the undetermined correction required by n. In the case of T, Ω, i, q, these corrections are extremely small, of the same order as the probable errors of the elements, so that we may regard all these as accurately known. But any variation of n involves a strictly corresponding variation of a and e. We cannot vary a and keep n the same, as Mr. Innes appears to suggest. Hence it still seems to me that the value of n given by him does involve a change of 18·8 days in the period.

The question really at issue is, I take it, the nature of osculating ellipses. It seems to me simpler to determine these with reference to the Sun's action alone; for we cannot take the whole planetary action into account consistently with elliptical motion, and the portion of their action that is consistent with elliptical motion is not a constant, but varies rapidly from point to point. But if a consistent theory and method are built up on the lines that Mr. Innes suggests, I am quite ready to admit that that course may have advantages. The whole subject of osculating orbits seems to deserve rather fuller treatment than is sometimes given to it.

Measures of the Proper Motion Star βGC 6869.
By S. W. Burnham, M.A., Assoc. R.A.S.

The large proper motion of this wide pair of faint stars was detected by Bellamy in measuring the Oxford photographic plates. It is given as No. 6869 in my *General Catalogue of Double Stars*, as the component stars are moving together, and doubtless form a physical system. The principal component appears to be B.D. $+24^{\circ}$ 2733 ($9^{m}\cdot0$). The other has nearly the same magnitude, and the two are moving in space at the same rate. The measures have been continued since 1905, and should now give an accurate value for the common proper motion of A, B. In part ii., p. 655, I have given earlier relations of A, C from meridian and photographic sources, but as these have much larger errors than careful and repeated measures with the micrometer, they may be disregarded in finding the proper motion.

The following are the mean results of the several sets of measures connecting A with the $9^{m}\cdot0$ star, B.D. $+24^{\circ}$ 2734 :—

A and C.

1903·02	120°·6	182″·30	4n
1904·32	120·2	180·57	2n
1905·42	120·0	179·34	3n
1906·18	119·9	178·56	6n
1907·25	119·6	177·30	4n
1909·15	119·4	175·05	3n
1910·17	119·2	173·56	3n
1911·17	118·9	172·31	4n

From these positions the most probable value of the proper motion of A is

$$1''\cdot348 \text{ in } 145^{\circ}\cdot2.$$

With reference to the question of any motion in the comparison star, I have connected with it a small star, nearly if not quite as faint as $13^{m}\cdot5$:—

C and *a*.

1906·21	181°·8	55″·91	3n
1907·20	181·6	55·61	3n
1909·14	181·3	55·51	2n
1910·24	181·7	55·63	3n
1908·20	181·6	55·66	11n

It is safe to say that C has no sensible movement, and that the proper motion given for A is correct, subject only to the unavoidable errors of micrometric measures.

These stars furnish a fine example of a double of the 61 Cygni type, where the components are moving in space with nearly the same rate and direction. The following are the mean results of my measures of these stars :—

A and B.

1903·02	74°·7	45″·21	4n	*β*
1904·31	74·5	45·02	3n	*β*
1905·42	74·1	45·22	2n	*β*
1906·12	74·1	45·19	3n	*β*
1907·18	74·3	45·21	3n	*β*
1911·18	74·1	45·19	3n	*β*

So far, there is no apparent relative change. It may be that, when the measures have been continued for a long time, it will appear that there is a small fraction of a degree of difference in the direction of the two stars, as in 61 Cygni, but it is certain that they form a system of some kind.

I have given in part ii. several positions derived from meridian and photographic observations, which are in fair accord with the direct measures, but of course subject to a larger error. In part ii., p. 1076, *for* 1″·696 *read* 1″·382, this being the value from the measures of A, D.

Yerkes Observatory:
1911 *March* 21.

Measures of a small Star with a large proper Motion.

By S. W. Burnham, M.A., Assoc. R.A.S.

In *Monthly Notices* for May 1908, lxviii. p. 521, I have given the results of measures to that time of a small star having an extraordinary proper motion. This movement was detected by Dr. Max Wolf from an examination of photographic plates made in 1892 and 1906. I compared this star in 1907 and 1908 with another star in the field, and with these positions and that from the photographic print of the negative made in 1892 deduced the proper motion of the star in question,

$$1''\cdot225 \text{ in } 190^\circ\cdot2.$$

Similar observations of a third very faint star measured from B made it certain that the latter star had no sensible movement of its own, and that the proper motion found was the true movement of the $9^{m}\cdot7$ star A.

The measures of A, B have been continued each year since the dates given, and it is safe to say that the proper motion of this small star is as accurately known as that of most of the naked-eye stars after meridian observations covering the best part of a century.

The following are the mean results of the several sets of measures of this time:—

A and B.

1892·23	242°·1	91″·47		Photo
1907·03	252·10	81·54	4n	β
1908·15	253·10	80·88	4n	β
1909·12	253·81	80·25	4n	β
1910·10	254·55	79·92	3n	β
1911·07	255·33	79·06	3n	β

From all the measures, the most probable value of the proper motion of A is

$$1''{\cdot}228 \text{ in } 190°{\cdot}4.$$

The place (1900) of this star is

$$\left.\begin{array}{l}\text{R.A. } 11^h\ 23^m\ 20^s \\ \text{Decl. } +8°\ 6'\ 1''\end{array}\right\}$$

It is easily found from the $7^m{\cdot}5$ star B.D. $+8°$ 2512, which is 70^s following, and $185''$ south.

Yerkes Observatory:
1911 *March* 21.

Observations of Nebulæ made at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

At the request of Dr. Dreyer, in order to identify certain nebulæ observed by Sir W. Herschel in his sweep of April 2, 1810, this region was surveyed photographically with the 30-inch reflector.

The zone extended from R.A. $9^h\ 30^m$ to $12^h\ 0^m$ and Dec. $74°\ 20'$ to $78°\ 20'$, and was covered by 10 photographs, with an exposure of 90 minutes on each.

In addition to Herschel's nebulæ in question, all of which were identified, a number of previously unknown nebulæ were found on the plates. That these images were really of nebulæ was verified by reference to the photograph of this region taken by Mr. Franklin Adams with a 10-inch lens, with an exposure of 2 hours. They are generally very faint (16 to 17 magnitudes), and small.

The plates were measured with the astrographic micrometer. For determining the constants, six stars were measured on each plate, their positions being derived from the Greenwich Astrographic Catalogue (now in preparation).

Except in a few cases of very indefinite nebulæ, the positions derived from the photographs should not be more than $1''$ in error.

The abbreviations used in the summary description are the same as in the N.G.C., except that the brightness is indicated by the estimated magnitude.

Mean Places of Nebulæ.

No.	No. in Herschel.	R.A. 1900·0. h m s	Annual Precession in R.A.	Dec. 1900·0. ° ′ ″	Annual Precession in Dec.	Remarks.
1	III. 963	9 27 42·35	+6·5766	+76 45 55·2	−15·780	
2		9 33 47·42	6·4676	76 48 25·0	16·103	16, *v*S, R.
3	I. 282	9 33 48·80	6·1088	75 18 53·3	16·105	
4		9 39 40·49	6·3060	76 38 16·1	16·404	17, *vv*S, ?F✳.
5		9 43 15·88	6·2025	76 30 40·4	16·583	17, *vv*S, ?F✳.
6	II. 903	9 46 17·33 16·23	6·1010	76 20 13·1 14·2	16·730	
7	III. 966	10 4 26·57	6·1932	78 18 43·9	17·552	
8	III. 964	10 6 44·59	5·3915	74 42 50·8	17·648	
9	III. 965	10 8 54·41	5·3715	74 50 41·0	17·736	
10		10 12 20·36	6·0039	78 20 23·8	17·874	17, *v*S.
11	I. 283	10 13 14·03	5·2634	74 40 41·8	17·910	
12		10 21 54·15	5·2185	75 30 28·9	18·238	17, S, R.
13	III. 316	10 26 17·94	4·9597	74 16 41·7	18·394	
14		10 33 55·93	5·3817	78 1 0·7	18·649	17, *c*S, R.
15	I. 284	10 36 13·94	5·1975	77 19 58·5	18·722	
16		10 36 33·31	5·1691	77 12 41·7	18·732	17, *p*S, *v* dif.
17		10 36 41·21	5·0864	76 43 27·3	18·736	15½, *v*S.
18		10 37 29·08	5·2186	77 37 37·3	18·761	17, *vv*S, *lb*M.
19		10 39 35·37	5·1228	77 22 24·1	18·825	16, E 45′ 100°, ?Spiral.
20		10 40 23·22	5·0959	77 19 42·9	18·849	16, E 50″ 170°, ?Spiral.
21		10 42 32·95	5·0653	77 27 56·5	18·912	15, *v*S, C, R.
22		10 42 57·43	5·0325	77 19 28·4	18·924	15½, *l*E 180°, stell N.
23		10 43 23·44	5·0418	77 26 52·5	18·936	15½, E 30″ 180°, stell N.
24		10 46 10·48	4·6896	75 20 19·1	19·015	17, *vv*S, R.
25		10 46 28·49	4·5639	74 13 16·6	19·023	16, E 60″ 70°.
26		10 48 29·31	4·6441	75 22 14·3	19·078	17, *vv*S, R.
27	III. 967	10 52 1·53	4·6075	75 43 37·7	19·171	
28		10 52 37·90	4·8137	77 27 52·9	19·186	16½, E 15″ 85°.
29	III. 968	10 54 27·12	4·5556	75 44 18·1	19·232	
30	II. 904	10 55 45·39	4·5180	75 39 12·3	19·263	
31		10 57 1·51	4·3795	74 29 51·0	19·294	16, S, R.
32		10 58 26·41	4·4378	75 26 18·9	19·327	17, *c*S, R.
33		10 59 51·19 50·95	4·6028	77 14 8·3 6·5	19·360	15½, *v*S, R.
34	d'Arrest	11 4 27·80	4·3692	76 6 44·0	19·460	D✳ (see note).
35		11 6 1·93	4·4757	77 28 26·3	19·493	16, *vv*S, R.
36	II. 905	11 26 19·30	3·8120	75 10 46·9	19·830	
37	III. 969	11 26 19·64	3·7988	74 55 49·0	19·830	
38	*h* 917	11 30 9·21	3·7603	75 50 24·7	19·876	
39	III. 970	11 36 48·98 48·64	3·7034	77 55 44·3 44·3	19·944	
40	III. 971	11 43 44·04	+3·4225	+74 51 25·4	−19·996	

Notes.—Nos. 6, 33 and 39 were each measured on two plates and the separate results are given.

N.G.C. 2938 = *h* 612. R.A. $9^h\ 34^m\ 48^s$. Dec. $+77^\circ\ 1'$. There is a faint star here but no nebulosity. The position is clearly indicated by the double star mentioned in the description.

N.G.C. 3061 = *h* 653. R.A. $9^h\ 52^m\ 42^s$. Dec. $+76^\circ\ 37'$. Nothing shown on the photograph.

N.G.C. 3484 = *h* 802. R.A. $10^h\ 55^m\ 29^s$. Dec. $+76^\circ\ 21'$. Nothing shown on the photograph.

N.G.C. 3538 = d'Arrest = No. 34 above. Doubtful. Appears as a double star on the photographs. P 310°, d 12″, mags. 17, 17.

N.G.C. 3752 = No. 38 above = *h* 917, but not H. 905 = No. 36 above.

Royal Observatory, Greenwich:
1911 *April* 7.

On the Variable RW Cassiopeiæ.

By E. T. Whittaker, Sc.D., F.R.S., and C. Martin. (Plate 12.)

The variability of this star, which is B.D. + 57°342, was discovered in 1905 by Madame Ceraski.* The period was found by Blažko † to be 14·80 days.

The light-curve discussed in the present paper has been obtained from ninety-eight plates taken with the 15-inch photographic reflector of Dunsink Observatory between 1909 September and 1911 March. The magnitude m has been deduced from measures of the diameter D of the stellar image, in accordance with the following law:—If, corresponding to any star, a point be taken whose ordinate is m and whose abscissa is $D^{\frac{2}{3}}$, the points so obtained for stars on the same plate lie on a straight line. This law, which was deduced empirically from measurements made on plates of the Pleiades and the χ Persei cluster, satisfies the Dunsink plates better than any of the other empirical laws which have been proposed at various times, *e.g.* that m is a linear function of D, or of $D^{\frac{1}{2}}$, or of log D, or that $1/m$ is a linear function of D.

In order to determine the straight line corresponding to each plate, it is necessary to have on the plate comparison stars of known magnitude. These were obtained by photographing the region of RW Cassiopeiæ on the same plates with the cluster

* *Ast. Nach.*, No. 4010.

† *Ast. Nach.*, No. 4108 (1906). Further observations have been published by Enebo, *Ast. Nach.*, No. 4207 (1907); by Seares, *Laws Bulletin*, No. 10 (1907); and by Prăcka, *Ber. des Ast. Obs. Nižbor*, Ser. I., No. 1 (1909).

χ Persei, at the same altitude and on the same night. The straight lines corresponding to these plates were determined from the known magnitudes of the χ Persei stars as given by Schwarzschild,* and the magnitudes of seven stars in the neighbourhood of RW Cassiopeiæ were thus derived. The results were as follows:—

Comparison Stars for RW Cassiopeiæ.	Photographic Magnitude.
B.D. +57° 346	8·79
+56 301	8·99
+57 355	9·51
+57 359	9·71
+57 337	9·85
* near B.D. +57 346	10·36
* near B.D. +57 337	10·93

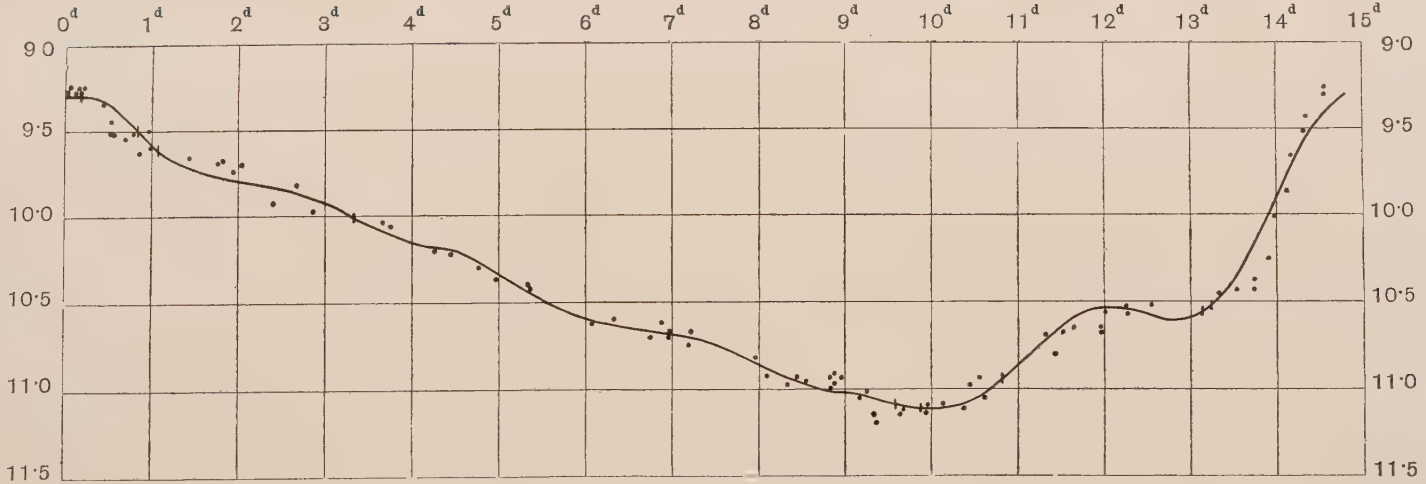
The magnitudes of the comparison stars having been thus obtained, the magnitude of the variable on any plate was determined in the following way. On accurately squared paper, a point was marked corresponding to each comparison star, the abscissa being proportional to $D^{\frac{3}{2}}$, as obtained by measurement of the image of this star on the plate, and the ordinate being proportional to its magnitude. A straight line was then drawn so as to pass as nearly as possible through all the seven points thus obtained: the average value of the deviation of one of these points from the line was found to correspond to an error of 0·04 mag., which may be taken as an indication of the degree of accuracy of the work. The diameter D of the image of the variable being measured, the magnitude of the variable is then given as the ordinate of that point on the line whose abscissa is $D^{\frac{3}{2}}$.

The magnitudes of the variable RW Cassiopeiæ, as deduced by this method from ninety-eight plates, are given in the following table. The first column gives the Greenwich mean time of the exposure, the second column the date in Julian days and decimals, and the third column the interval from this to J.D. 2418586·36, which was taken as the zero of phase. The fourth column gives the phase when the period is assumed to be 14·81 days: this period satisfies the Dunsink observations best. As it is known that the period of some short-period variables is subject to slight progressive changes, it seemed desirable to use a period derived exclusively from our own observations. The fifth column gives the magnitude of the variable, and the sixth column contains remarks on the quality of the star images on the plates.

* *Public. der v. Kuffner'schen Sternwarte*, V (1900) C.

Date G.M.T.				Julian Day.	Days from Origin.	Phase.	Magnitude.	Images.
1909.				2418				
Sept.	7	12	20	557·54	− 28·82	0·80	9·52	Good.
	10	11	29	560·50	− 25·86	3·76	10·06	Good.
	13	11	2	563·48	− 22·88	6·74	10 71	Fair.
	17	12	39	567·55	− 18·81	10·81	10·94	Bad.
	20	10	59	570·48	− 15·88	13·74	10·43	Fair.
	22	9	41	572·43	− 13·93	0·88	9·63	Good.
	24	9	37	574·42	− 11·94	2·87	9·97	Fair.
Oct.	5	10	16	585·45	− 0·91	13·90	10·25	Good.
	6	8	0	586·36	0·00	0·00	9·12	Good.
	8	8	59	588·40	+ 2·04	2·04	9·70	Good.
	11	7	36	591·34	4·98	4·98	10·37	Good.
	12	9	56	592·44	6·08	6·08	10 63	Good.
	13	7	22	593·33	6·97	6·97	10·71	Good.
	18	8	22	598·37	12·01	12·01	10·56	Fair.
	20	7	54	600·35	13·99	13·99	10·01	Fair.
	21	7	44	601·35	14·99	0·18	9·30	Fair.
	25	8	4	605·36	19·00	4·19	[10·00]	Bad.
	28	8	46	608·39	22·03	7·22	[10·57]	Bad.
	29	8	28	609·38	23·02	8·21	[10·55]	Bad.
	30	11	9	610·49	24·13	9·32	[10·93]	Bad.
	31	6	42	611·30	24·94	10·13	11·10	Good.
Nov.	3	6	36	614·30	27·94	13·13	10·55	Fair.
	6	9	10	617·41	31·05	1·43	9·66	Good.
	7	8	28	618·38	32·02	2·40	9·93	Good.
	9	9	28	620·42	34·06	4·44	10·22	Good.
	10	6	41	621·30	34·94	5·32	10·40	Good.
	14	7	28	625·34	38·98	9·36	11·20	Fair.
	14	12	46	625·56	39·20	9·58	11·10	Fair.
	15	8	14	626·37	40·01	10·39	11·12	Good.
	15	13	20	626·59	40·23	10·61	11·06	Fair.
	16	6	56	627·31	40·95	11·33	10·70	Good.
	20	8	17	631·37	45·01	0·58	9·53	Fair.
	23	10	29	634·46	48·10	3·67	10·04	Bad.
	29	9	24	640·42	54·06	9·63	11·15	Good.
	30	7	34	641·34	54·98	10·55	10·94	Good.
Dec.	3	12	13	644·53	58·17	13·74	10·37	Good.
	4	7	55	645·35	58·99	14·56	9·30	Fair.
	20	9	2	661·40	75·04	0·99	9·60	Fair.
	31	8	56	672·39	+ 86·03	11·98	10·68	Good.

Date G.M.T.				Julian Day.	Days from Origin.	Phase.	Magnitude.	Images.
1910.				2418				
Jan.	11	7	9	683·32	+ 96·96	8·10	10·93	Good.
	12	7	27	684·33	97·97	9·11	[11·23]	Bad.
	12	12	36	684·55	98·19	9·33	11·15	Good.
	31	12	44	703·55	117·19	13·52	10·43	Fair.
Feb.	2	11	8	705·49	119·13	0·65	9·52	Good.
Mar.	10	9	54	741·44	155·08	6·98	10·68	Fair.
	10	15	12	741·66	155·30	7·20	10·75	Bad.
	11	9	48	742·43	156·07	7·97	10·83	Good.
	15	9	56	746·44	160·08	11·98	10·65	Fair.
	18	9	39	749·43	163·07	0·16	9·26	Fair.
	21	13	37	752·59	166·23	3·32	10·02	Fair.
	30	12	3	761·53	175·17	12·26	10·57	Fair.
	31	11	39	762·51	176·15	13 24	10·52	Fair.
	31	14	7	762·61	176·25	13·34	10·46	Fair.
Apr.	1	8	39	763·38	177·02	14·11	9·86	Fair.
	1	10	29	763·46	177·10	14·19	9·66	Bad.
	1	13	20	763·58	177·26	14·31	9·52	Fair.
	7	9	51	769·43	183·07	5·35	10·42	Fair.
	8	9	0	770·40	184·04	6·32	10·60	Fair.
	10	9	19	772·41	186·05	8·33	10·98	Fair.
	10	11	30	772·51	186·15	8·43	10·94	Fair.
	10	14	21	772·62	186·26	8·54	10·96	Fair.
	13	13	33	775·59	189·23	11·51	10·68	Good.
	16	9	49	778·43	192·07	14·35	9·43	Fair.
	16	15	3	778·65	192·29	14·57	9·26	Fair.
	28	12	25	790·54	204·18	11·65	10·65	Fair.
	29	10	0	791·44	205·08	12·55	10·52	Fair.
May	3	10	32	795·46	209·10	1·76	9·69	Fair.
	3	11	40	795·51	209·15	1·81	9·68	Fair.
	3	14	45	795·64	209·28	1·94	9·74	Good.
	6	10	39	798·47	212·11	4·77	10·30	Fair.
	10	11	59	802·52	216·16	8·82	11·00	Good.
	10	13	8	802·57	216·21	8·87	10·98	Fair.
Sept.	29	10	57	944·48	358·12	2·68	9·82	Good.
Nov.	2	6	44	978·30	391·94	6·88	10·62	Good.
	2	14	50	978·64	392·28	7·22	10·67	Fair.
	4	6	39	980·30	393·94	8·88	10·91	Good.
	4	8	31	980·38	394·02	8 96	10·94	Good.
	4	13	22	980·58	+ 394·22	9·16	11·05	Good.



Observed magnitudes of RW Cassiopeiæ, compared with the curve whose equation is

$$m = 10{\cdot}36 + 0{\cdot}70\sin(\theta + 233^\circ) + 0{\cdot}17\sin(2\theta + 260^\circ) + 0{\cdot}08\sin(3\theta + 234^\circ) + 0{\cdot}13\sin(4\theta + 249^\circ) + 0{\cdot}09\sin(5\theta + 280^\circ) + 0{\cdot}06\sin(6\theta + 304^\circ).$$

Period used, 14·81 days. Observations falling exactly on the curve are denoted by strokes.

Date G.M.T.				Julian Day.	Days from Origin.	Phase.	Magnitude.	Images.
1910.				2418				
Nov.	4	15	33	980·67	+ 394·31	9·25	11·02	Good.
	5	6	43	981·30	394·94	9·88	11·11	Good.
	5	8	39	981·38	395·02	9·96	11·10	Fair.
	11	6	52	987·31	400·95	1·08	9·62	Fair.
				2419				
	29	6	40	005·30	418·94	4·26	10·20	Good.
Dec.	20	6	27	026·29	439·93	10·44	10·98	Bad.
	21	6	18	027·29	440·93	11·44	10·81	Fair.
1911.								
Jan.	2	11	11	039·49	453·13	8·83	10·93	Fair.
	9	6	58	046·31	459·95	0·84	9·51	Fair.
	9	10	1	046·44	460·08	0·97	9·50	Fair.
	18	9	1	055·40	469·04	9·93	11·14	Fair.
	23	6	35	060·30	473·94	0·02	9·28	Good.
	23	7	29	060·34	473·98	0·06	9·25	Good.
	23	9	21	060·41	474·05	0·13	9·27	Good.
	23	10	17	060·45	474·09	0·17	9·27	Good.
	23	11	8	060·49	474·13	0·21	9·25	Good.
Feb.	19	7	40	087·34	500·98	12·25	10·53	Good.
	22	7	39	090·34	503·98	0·44	9·35	Good.
	22	9	29	090·42	504·06	0·52	9·45	Good.
Mar.	9	9	11	105·41	+ 519·05	0·70	9·55	Fair.

The results (excepting those marked by brackets, which were rejected as untrustworthy) are plotted in the annexed figure (plate 12). The deep bay in the ascending branch of the light-curve is a somewhat unusual feature in stars of this class, though bays in the descending branch are not uncommon.

The range of brightness is much greater in photographic than in visual observations of this star.

The light-curve thus determined has been analysed into its harmonic constituents according to Fourier's theorem. If θ denote the time, measured in angle at the rate of 360° to a complete period, and reckoned from the origin previously assumed for the phase, the photographic magnitude is thus found to be represented by the formula

$$m = 10{\cdot}36 + 0{\cdot}70 \sin(\theta + 233^\circ) + 0{\cdot}17 \sin(2\theta + 260^\circ) + 0{\cdot}08 \sin(3\theta + 234^\circ) + 0{\cdot}13 \sin(4\theta + 249^\circ) + 0{\cdot}09 \sin(5\theta + 280^\circ) + 0{\cdot}06 \sin(6\theta + 204^\circ).$$

It will be noticed that the third harmonic is smaller than the fourth or fifth.

The values of m calculated from this formula, at intervals of 10°, are as follows:—

θ.	Phase in Days.	m.	θ.	Phase in Days.	m.
°			°		
0	0	9·30	180	7·41	10·73
10	0·41	9·31	190	7·82	10·82
20	0·82	9·48	200	8·23	10·92
30	1·23	9·66	210	8·65	10·99
40	1·65	9·76	220	9·06	11·03
50	2·06	9·80	230	9·47	11·07
60	2·47	9·83	240	9·88	11·11
70	2·88	9·90	250	10·29	11·10
80	3·29	10·00	260	10·70	10·99
90	3·71	10·10	270	11·12	10·81
100	4·12	10·17	280	11·52	10·63
110	4·53	10·21	290	11·94	10·54
120	4·94	10·32	300	12·35	10·56
130	5·35	10·44	310	12·76	10·61
140	5·76	10·55	320	13·17	10·55
150	6·18	10·62	330	13·59	10·29
160	6·59	10·65	340	13·99	9·89
170	7·00	10·68	350	14·40	9·49

The curve represented by the harmonic formula is drawn in the figure: observations falling exactly on the curve are denoted by short strokes.

Dunsink Observatory,
Castleknock, Ireland:
1911 *April* 3.

Note on the Variable 97. 1910 *Cygni.*

By Arthur R. Hinks, M.A.

This star appeared on three plates taken with the Sheepshanks Equatorial of the Cambridge Observatory on three days in 1909 August, and was missing on three plates of the same region taken 1910 August. A brief announcement, with a diagram of the field, appeared in *A.N.*, 4448.

The place of the star, referred to the four stars 8796, 8805, 8815, and 8828 of the Lund A.G. catalogue, was

R.A. $19^h\ 49^m\ 55^s{\cdot}01$ Decl. $+36°\ 46'\ 57''{\cdot}4$ 1900·0,

and its provisional magnitude for 1909 August 10 was $10^m{\cdot}5$, deduced by photographic extrapolation from Lund magnitudes. The star is in the field of Kapteyn Selected Areas, Special List, No. 8, and this led to its detection.

Visual search for the star was made with the Sheepshanks Telescope (aperture 12-inch) on 1910 September 20, September 27, and November 21, but without result. On the latter night it must have been below $13\frac{1}{2}^m$.

In January I received a letter from Professor Barnard, dated 1911 January 3, sending me a diagram and measures of a number of faint stars in the field, of which two, Nos. 14 and 17 of his list, were close to my published place. The observations had been made with the 40-inch refractor during 1910 November and December.

I then measured all the faint stars on my early plates and compared them with Professor Barnard's diagram. It was evident that his star 14 was my variable. Its magnitude on 1910 November 24 was estimated by him at $16^m{\cdot}7$.

I have recently received several further letters from Professor Barnard, who has been so kind as to make observations immediately the region became visible again in the morning. On 1911 March 20 he writes: "I got your 'Nova' this morning; it is bright, and is doubtless only a variable. It is slightly yellowish: estimated magnitude 11. It must therefore go through at least five magnitudes. I made it 0·4 brighter than 22 [which had been assumed $= 12^m$]. I tried it for focus, and found that its out of focus image showed nothing peculiar." Again, on 1911 March 27, he writes: "This morning it was seen in a dense misty sky, 0·1 brighter than 22."

During the last fortnight the mornings have been persistently cloudy, and I have been unable to observe the star at Cambridge. Judging from Professor Barnard's last observation, it seems possible that the star is already diminishing again.

My measures of the faint stars about the variable were made on the two exposures of plate 896 (1909 August 7). Using the

four Lund stars mentioned above, I have reduced these stars to standard for 1900·0; the resulting coordinates with respect to the centre

$$\alpha\ 19^h\ 48^m\ 28^s{\cdot}00 \qquad \delta + 37^\circ\ 6'\ 0''{\cdot}0 \qquad (1900{\cdot}0)$$

are as follows, in minutes of arc :—

Barnard's No. and Mag.		ξ.	η.
1	13·0	+14′·803	−18′·467
4	12·8	+16·120	−19·302
5	13·5	+16·361	−17·043
8_1	14	+16·744	−18·389
13	13·5	+17·362	−16·326
22	12	+18·741	−18·576
25	13·3	+19·965	−19·984
26	13	+20·574	−18·986
. . . .		. . .	. . .
14 = Var.		+17·421	−19·011

From these figures we have

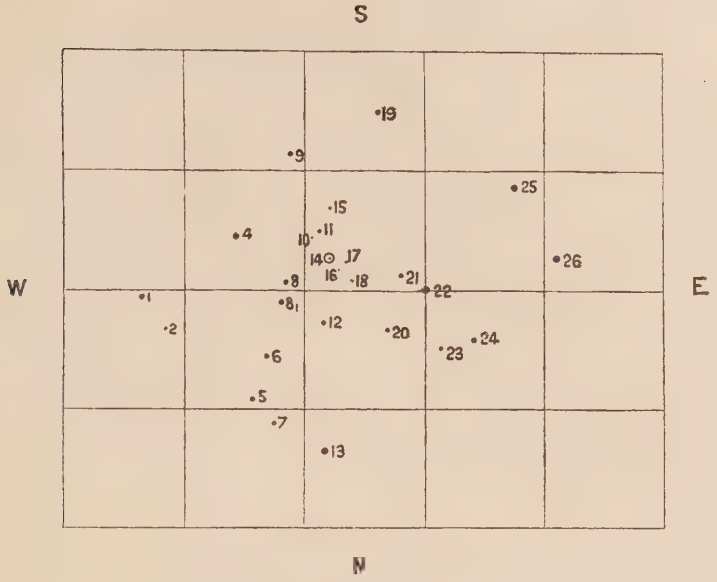
1909 Aug. *22 − Var.	$\Delta\xi$ + 1′·320	$\Delta\eta$ + 0′·435

From the measures which Professor Barnard has kindly sent me, I find approximately (without taking account of small corrections)

1910 Nov. *22 − Var.	$\Delta\xi$ + 1′·343	$\Delta\eta$ + 0′·449
1911 Mar. ...	+1·327	+0·427

There can be no doubt of the identity of the faint and difficult object of magnitude 16·7 measured by Professor Barnard in November last with the brighter star seen by him later, and photographed by me in 1909 August.

The accompanying diagram shows the stars measured as above on my photograph, together with the fainter stars measured with the micrometer on the 40-inch by Professor Barnard. The magnitudes are Barnard's.



Field of variable 97. 1910. Scale 1 square = 100″.

Brighter stars from photographic measures above; fainter stars from micrometric measures communicated by Professor Barnard.

Magnitudes from estimates by Barnard:

1	13·0	11	14·5	21	14
2	15	12	14·2	22	12
4	12·8	13	13·5	23	14
5	13·5	14	var.	24	13·5
6	13·8	15	14·5	25	13 3
7	14	16	v. faint	26	13
8_1	14	17	15		
8	14·5	18	v. faint		
9	13·5	19	13·7		
10	v. faint	20	14		

The star 22 is $0^m\ 42^s$ following and 68″ south of the $8^m{\cdot}9$ star B.D. + 36°3750 = Lund A.G. 8805.

I must express my sincere thanks to Professor Barnard for the interest and care which he has given to the star, and his kind permission to make use of the information which he has communicated in several letters.

Cambridge Observatory:
1911 *April* 11.

The Long-Period Variable U Persei (Ch. 678) in 1909–11.

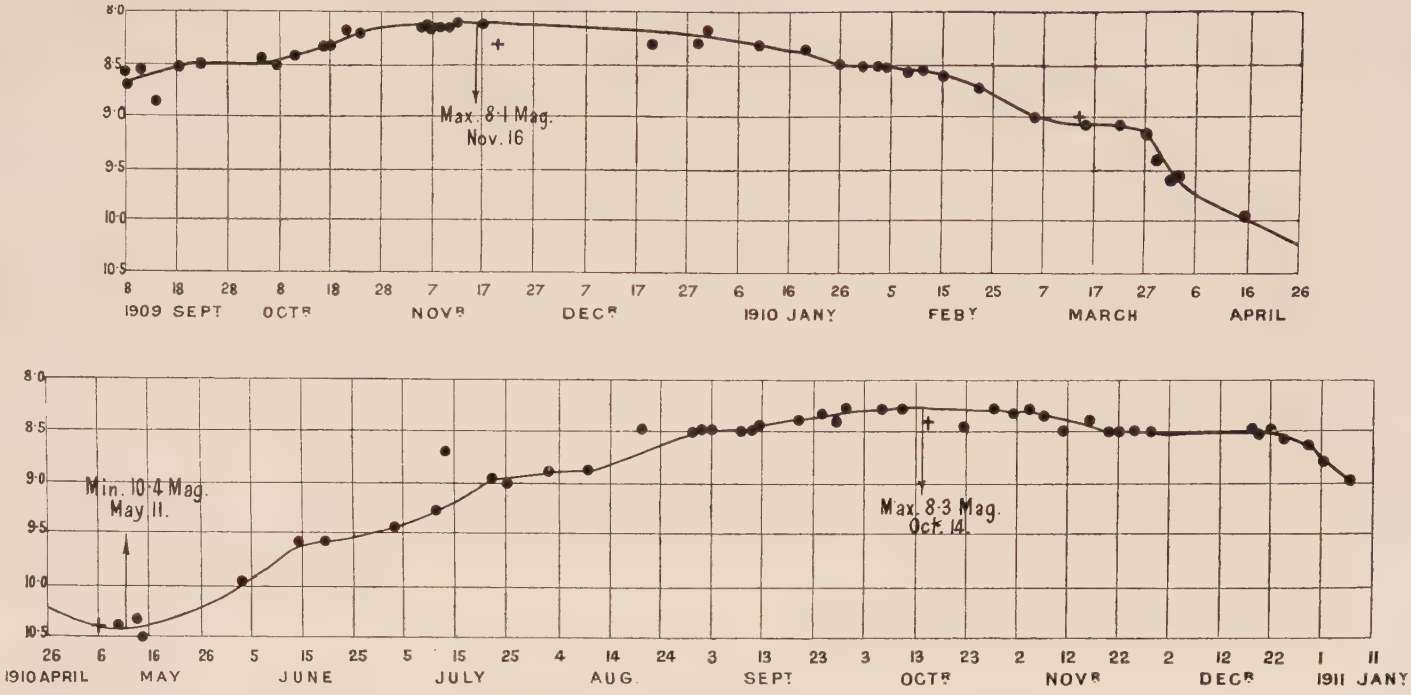
By A. N. Brown, M.A. (Plate 13.)

The eighty-nine observations of U Persei printed below have been arranged upon precisely the same plan as that followed in the case of my observations of RT Cygni published in *Monthly Notices*, lxxi. pp. 486-489. Every remark made respecting the latter as regards the contents of each column, the abbreviations employed, and the method of recording the comparisons applies here also, except that in these observations Hagen's magnitudes have been adopted throughout. These have been obtained from the Catalogue in Series III. of his *Atlas Stell. Var.*

Both the maxima shown on the curve (plate 13) are so much protracted that it is impossible to describe either as having occurred on any particular day. The dates given are approximately those when the middle of the maximum phase in each case occurred. It was far more easy to assign a definite day for the minimum.

The Observations.

Date.	Julian date.	Instr.	Class.	Comparisons.	Ded. mag.
1909.					
Sept. 8	8558	t. 30	1	4 − 3·5, 7 + 0·5	8·6
9	8559	,,	,,	6 − 1·5, 8 + 0·5 ; 7 − 1·5	8·7
11	8561	,,	,,	4 − 3·5, 7 + 1	8·5
14	8564	t. 60	,,	7 − 3, 11 + 2 ; 8 − 1	8·9
19	8569	T. 50	,,	4 − 3, 7 + 1	8·5
23	8573	t. 30	,,	4 − 2·5, 7 + 1	8·5
Oct. 5	8585	,,	,,	4 − 2, 7 + 1·5	8·4
8	8588	t. 60	,,	4 − 3, 7 + 1	8·5
11	8591	t. 30	,,	4 − 2, 7 + 1·5	8·4
17	8597	,,	,,	4 − 1, 7 + 2·5	8·3
18	8598	,,	,,	4 − 1, 7 + 2·5	8·3
21	8601	,,	,,	3 − 1·5, 4 + 0·5	8·2
24	8604	,,	,,	3 − 2, 7 + 4	8·2
Nov. 5	8616	,,	,,	3 − 1, 4 + 0·5	8·1
6	8617	,,	,,	3 − 1, 4 + 1	8·1
7	8618	,,	,,	4 + 0·5	8·2
8	8619	,,	,,	3 − 1, 4 + 0·5	8·1
10	8621	,,	,,	3 − 1, 4 + 0·5	8·1
12	8623	,,	,,	3 − 0·5, 4 + 1	8·1
17	8628	,,	,,	3 − 1, 4 + 1	8·1
20	8631	T. 50	2	4 − 1	8·3
Dec. 20	8661	T. 25	1	4 − 0·5, 5 + 1 ; 4 − 1, 7 + 3	8·3



LIGHT CURVE OF U PERSEI, 1909-11.—A. N. BROWN.

The Observations—continued.

Date.	Julian date.	Instr.	Class.	Comparisons.	Ded. mag.
1909.					
Dec. 29	8670	t. 30	1	4 − 1, 7 + 3	8·3
31	8672	,,	,,	3 − 1·5, 4 + 0·5 ; 4 + 1	8·1
1910.					
Jan. 10	8682	t. 30	1	4 − 1, 7 + 3	8·3
19	8691	,,	,,	4 − 1, 7 + 2·5	8·3
26	8698	,,	,,	4 − 2·5, 7 + 1	8·5
30	8702	,,	,,	4 − 3, 7 + 1	8·5
Feb. 2	8705	T. 25	,,	4 − 3, 7 + 1	8·5
4	8707	,,	,,	4 − 3, 7 + 1 ; 5 − 2·5, 6 + 1	8·5
8	8711	,,	,,	4 − 3·5, 7 + 1	8·5
11	8715	,,	,,	4 − 3, 7 + 1	8·5
15	8718	,,	,,	= 7	8·6
22	8725	,,	,,	7 − 1, 11 + 4 ; = 8	8·7
Mar. 5	8736	t. 60	,,	7 − 3·5, 11 + 1	9·0
14	8745	T. 50	2	7 − 4, 11 + 1	9·0
15	8746	,,	1	7 − 4, 11 + 1 ; 8 − 3·5, 13 + 1	9·1
22	8753	T. 95	,,	7 − 4·5, 11 + 1	9·0
27	8758	,,	,,	11 − 0·5, 12 + 0·5	9·2
29	8760	T. 50	,,	= 16	9·4
Apr. 1	8763	T. 95	,,	18 − 0·5, 19 + 0·5	9·6
2	8764	,,	,,	19 + 0·5	9·6
15	8777	t. 85	,,	21 − 2, 33 + 1·5	9·9
May 6	8798	T. 50	2	33 − 2·5, 37 + 1·5	10·4
9	8801	T. 95	1	33 − 2·5, 37 + 1·5	10·4
13	8805	T. 120	,,	33 − 2, 37 + 1·5	10·3
14	8806	,,	,,	33 − 3, 37 + 1 ; 42 + 1	10·6
June 3	8826	T. 95	,,	22 − 2·5, 33 + 1·5	10·0
14	8837	,,	,,	18 − 0·5, 19 + 0·5	9·6
19	8842	,,	,,	18 − 0·5, 19 + 0·5	9·6
July 3	8856	T. 50	,,	16 − 0·5, 18 + 0·5	9·5
11	8864	,,	,,	11 − 2, 16 + 1	9·3
13	8866	,,	,,	7 − 1	8·7
22	8875	,,	,,	7 − 4, 11 + 1	9·0
25	8878	,,	,,	7 − 4·5, 11 + 1	9·0
Aug. 2	8886	t. 60	,,	7 − 3, 11 + 2 ; 8 − 2·5, 13 + 2·5	8·9
10	8894	,,	,,	7 − 3, 11 + 2	8·9
20	8904	p. 26	,,	= 6 ; 7 − 1	8·5
30	8914	t. 30	,,	4 − 3·5, 7 + 1	8·5

The Observations—continued

Date.	Julian date.	Instr.	Class.	Comparisons.	Ded. mag.
1910.					
Sept. 1	8916	t. 30	1	4 − 3, 7 + 1	8·5
3	8918	,,	,,	4 − 3, 7 + 1	8·5
8	8923	,,	,,	4 − 3, 7 + 1	8·5
11	8926	,,	,,	4 − 3, 7 + 1·5	8·5
12	8927	,,	,,	4 − 2·5, 7 + 1·5	8·5
20	8935	T. 50	,,	4 − 2, 7 + 2	8·4
24	8939	,,	,,	4 − 1·5, 7 + 2·5	8·4
27	8942	,,	,,	4 − 2·5, 7 + 2	8·4
29	8944	t. 30	,,	4 − 1, 7 + 3	8·3
Oct. 6	8951	,,	,,	4 − 1, 7 + 3	8·3
10	8955	t. 60	,,	4 − 1, 7 + 3	8·3
15	8960	,,	2	4 − 2, 7 + 2	8·4
22	8967	T. 50	1	4 − 2·5, 7 + 1·5	8·5
28	8973	t. 60	,,	4 − 1	8·3
Nov. 1	8977	,,	,,	4 − 1·5, 7 + 3	8·3
4	8980	t. 30	,,	4 − 1, 7 + 3	8·3
7	8983	T. 50	,,	4 − 1·5, 7 + 2·5	8·4
11	8987	t. 60	,,	4 − 3, 7 + 1	8·5
16	8992	,,	,,	4 − 2, 7 + 2	8·4
20	8996	T. 50	,,	4 − 3, 7 + 1	8·5
22	8998	t. 60	,,	4 − 3, 7 + 1	8·5
25	9001	T. 50	,,	4 − 3·5, 7 + 1	8·5
28	9004	,,	,,	4 − 3, 7 + 1	8·5
Dec. 18	9024	,,	,,	7 + 1	8·5
19	9025	,,	,,	4 − 3·5, 7 + 1	8·5
22	9028	,,	,,	7 + 1	8·5
24	9030	t. 60	,,	= 7	8·6
29	9035	,,	,,	7 − 0·5	8·7
1911.					
Jan. 1	9038	t. 60	1	7 − 1·5, 11 + 3	8·8
6	9043	,,	,,	7 − 4, 11 + 1	9·0

The Comparison Stars used.

Hagen's Number.	Hagen's Magn.	Hagen's Number	Hagen's Magn.
3	8·0	16	9·4
4	8·2	18	9·5
6	8·5	19	9·6
7	8·6	21	9·7
8	8·7	22	9·7
11	9·1	33	10·1
12	9·2	37	10·5
13	9·2	42	10·8

Ludgrove, New Barnet, Herts:
1911 *March* 20.

Note on Mr. A. S. Eddington's moving Cluster of Stars in Perseus.
By T. W. Backhouse.

It does not appear to have been remarked that nine out of the sixteen stars in this cluster (see *Monthly Notices*, vol. lxxi. p. 44), namely, those from the second to the tenth, form a large part of one of the most noticeable naked-eye clusters in the northern hemisphere,—a cluster which, as Mr. Eddington points out, is narrow in comparison with its length. Three other of his stars form a somewhat irregular curved line continuing the naked-eye cluster, while another, namely, ρ Aurigæ, is also in this line, though at a considerable distance. These thirteen stars form the "chain" to which Mr. Eddington refers. The other three of his moving cluster are somewhat widely scattered from it, namely, 30 and ϵ Persei and 15 Camelopardi; and therefore, if they belong to the group, this is a case analogous to that of Sirius, belonging to the Ursa Major group, if it is ultimately proved to do so. The naked-eye cluster includes α and σ Persei, which two stars are not included in the moving cluster. Probably, however, some of the smaller stars of the naked-eye cluster will be found also to belong to the moving cluster.

Errata in Mr. Tebbutt's Observations of Halley's Comet, in the Monthly Notices *for January* 1911 :

Page 225, Comet's apparent R.A. on Dec. 3, *for* $4^h\ 19^m\ 41^s{\cdot}22$ *read* $4^h\ 19^m\ 45^s{\cdot}22$.

„ Reduction to App. R.A. on June 5, *for* $-0^s{\cdot}08$ *read* $+0^s{\cdot}08$.

Errata in Dr. de Sitter's paper on the Principle of Relativity, in Monthly Notices *for March* 1911 :

Page 391, line 3 from bottom } *for t read ct.*
„ 395, „ 19 }

„ 403, „ 5 from bottom, *for* S_3 *read* T_1.

„ 404, „ 5 *for* $(1+\frac{r}{p}\sin v)$ *read* $(1+\frac{r}{p})\sin v$.